# Quantized Vershik–Kerov Theory and $q$-deformed Gelfand–Tsetlin Graph

Ryosuke Sato

Graduate School of Mathematics, Kyushu university, Fukuoka, Japan
E-mail : ma217052@math.kyushu-u.ac.jp

## Abstract

We propose natural quantized character theory for inductive systems of compact quantum groups. Also, we provided a $q$-deformation of the approximation theorem for ordinary characters of group due to Vershik–Kerov. This relate to Gorin's analysis on $q$-Gelfand–Tsetlin graph explicitly when the given quantum groups are quantum unitary groups.

**character theory of inductive groups**

**probability theory on branching graphs**

$\chi : G_\infty \to \mathbb{C}$ ; character

positive-type,
central,
normalized

$\chi|_{G_N} = \sum_\rho \mathbb{P}_N(\rho)\chi_\rho$ ⟶

$(\mathbb{P}_N)_N$ ; coherent system

($\mathbb{P}_N$ ; probability on $\widehat{G}_N$
with a certain relation)

↓

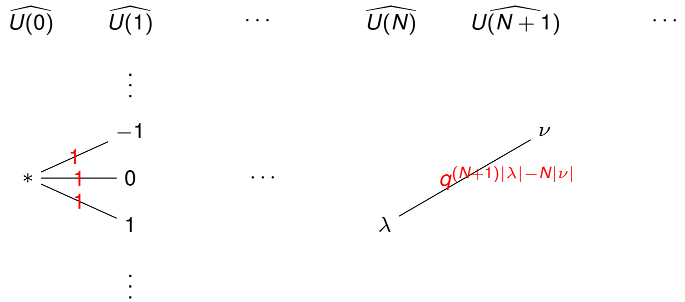
$\mathbb{P}$ ; central measure

(probability on the path space
with a certain invariance)

A character $\chi$ is extremal (="irreducible") if and only if the corresponding central probability measure $\mathbb{P}$ is ergodic with respect to the group of finite permutations of paths.

# $q$-Deformed Gelfand–Tsetlin Graph (Gorin, 2012)

the $q$-Gelfand–Tsetlin graph = the Gelfand–Tsetlin graph + the weights on edges




$\lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N)$ joins $\nu = (\nu_1 \geq \nu_2 \geq \cdots \geq \nu_{N+1})$ by an edge if $\nu_1 \geq \lambda_1 \geq \nu_2 \cdots \geq \nu_N \geq \lambda_N \geq \nu_{N+1}$.

There exists $q$-coherent systems and $q$-central measures which are q-deformations for coherent systems and central measures on the Gelfand–Tsetlin graph.

# The $q$-Deformation v.s. Character theory of CQGs

**Compact quantum group(=CQG)** is a $q$-deformation of the ring $C(G)$ of continuous functions on a compact group $G$.

We can define characters on CQGs by quite natural way.
They are called qauntized characters.

The $U_q(N)$ and the $U(N)$ have the same representation theory. In particular, the inductive system of $U_q(N)$ has the same branching graph : the Gelfand–Tsetlin graph.

We can find the correspondence quantized characters of inductive system of $U_q(N)$ and $q$-central measures on the Gelfand–Tsetlin graph.

By using the correspondence between quantized characters and “quantized” probability measures, we have the followings.

**Approximation theorem** (S. 2018):
For every extremal quantized character $\chi$ of an inductive system of compact quantum group there exists a sequence $\pi_1 \prec \pi_2 \prec \cdots$ such that $\pi_N \in \widehat{G_N}, \quad \pi_N \subset \pi_{N+1}|_{G_N}$ and

$$\chi|_{G_N} = \lim_{\substack{L\to\infty \\ L\geq N}} \chi_{\pi_L}|_{G_N},$$

where $\chi_{\pi_L}$ is the irreducible quantized character of the representation $\pi_L$.

**Boundary theorem** (Gorin 2012, S. 2018):
The set of extremal quantized characters of the inductive system of quantum unitary groups $U_q(N)$ (and ergodic $q$-central measures on $q$-Gelfand–Tsetlin graph) are parametrized by

$$\{\theta = (\theta_i)_{i=1}^{\infty} \in \mathbb{Z}^{\infty} \mid \theta_1 \leq \theta_2 \leq \cdots\}.$$

**Reference**:


V. Gorin, *The q-Gelfand–Tsetlin graph, Gibbs measures and q-Toeplitz matrices*, Adv. Math **229** (2012), no. 1, 201–266

R. Sato, *Quantized Vershik–Kerov theory and quantized central measures on branching graphs*, arXiv:1804.02644