Recent Developments for the Singular Values of Skew-Symmetric Gaussian Random Matrices

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$\mathcal{A}$: The space of $p \times p$, real, skew-symmetric matrices.

$A = (a_{ij}) \in \mathcal{A}$: A noncentral Gaussian random matrix with p.d.f.

$$f(A) = (2\pi)^{-p(p+1)/4} \exp\left[-\tfrac{1}{4} \operatorname{tr} (A - M)(A - M)'\right],$$

where $M = E(A)$.

The singular values of A: $\sigma_1 > \cdots > \sigma_q > 0$

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$D_\sigma = \begin{cases} \sigma_1 J \oplus \cdots \oplus \sigma_q J, & \text{if } p \text{ is even, } p = 2q \\ \sigma_1 J \oplus \cdots \oplus \sigma_q J \oplus 0, & \text{if } p \text{ is odd, } p = 2q+1 \end{cases}$$

Kuriki (2010) considered the singular value decomposition:

$$A = H D_\sigma H', \quad \text{where} \quad H \in SO(p).$$

The motivation: Problems in mathematical statistics, and a statistical analysis of a Japanese league's baseball scores.

Kuriki was led to Harish-Chandra's integral for $SO(p)$:

$$I_p(\sigma, \nu) = \int_{SO(p)} \exp\left(\tfrac{1}{2} \operatorname{tr}\ H D_\sigma H' D'_\nu\right) \, \mathrm{d}H$$

Note the remarkable connection:

Baseball scores $\longleftrightarrow$ Harish-Chandra's integral!

My poster will raise open problems concerning the *total positivity* properties of $I_p(\sigma, \nu)$.