Eigenvector Distribution and QUE for Deformed Wigner Matrices

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Random matrices and their applications
Kyoto University

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  We will consider the model:

$$D + \sqrt{t}W$$

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Eigenvectors	Supported on $\mathcal{O}(1)$ entries	Non-ergodic Delocalized	Completely Delocalized

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Thank you!