

# **PROBABILITY AND GEOMETRY IN HIGH DIMENSIONS**

**Laboratoire d'Analyse et de  
Mathématiques Appliquées (LAMA)**

**Université Paris-Est Marne-la-Vallée**

**May 17-21, 2010**

## **Organizers**

**Djalil Chafai**

**Olivier Guédon**

**Guillaume Lécué**

**Alain Pajor**





Dear Participants,

The aim of this workshop is to reflect on recent developments in Probability and Geometry in High Dimensions with emphasis on interactions with other fields of mathematics such as compressed sensing, sparse statistical problems, random matrices, and empirical processes.

We wish you a pleasant stay at Université Paris-Est Marne-la-Vallée.

The organizers.



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## 1 Programme

**Monday, 17.05.2010**

09:45-10:25 *Registration and coffee*

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10:25-10:30 *Opening*

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10:30-11:10 **Vladimir Koltchinskii**

Sparse Recovery in Linear Spans and Convex Hulls of Infinite Dictionaries

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11:10-11:40 *Coffee break*

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11:40-12:20 **Jared Tanner**

Random matrix theory and stochastic geometry in compressed sensing

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12:25-13:05 **Omer Friedland**

Random embedding of  $\ell_p^n$  into  $\ell_r^N$

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13:05-14:30 *Lunch break*

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14:30-15:10 **Philippe Jaming**

Some annihilating pairs in harmonic analysis

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15:10-15:40 *Coffee break*

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15:40-16:20 **Francis Bach**

High-Dimensional Non-Linear Variable Selection

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16:25-17:05 **Radosław Adamczak**

Geometric properties of random matrices with independent log-concave rows/columns

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**Tuesday, 18.05.2010**

9:45-10:25 **Leonid Pastur**

Central Limit Theorem for Linear Eigenvalue Statistics of Random Matrices with Independent Entries

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10:30-11:10 **Alexandre Tsybakov**

Estimation of High-Dimensional Low Rank Matrices

---

11:10-11:40 *Coffee break*

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11:40-12:20 **Rafał Łatała**

On 1-symmetric logarithmically concave distributions

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11:25-13:05 **Ionel Popescu**

Random Matrices and Analyticity of the Planar Limit

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13:05-14:30 *Lunch break*

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**Wednesday, 19.05.2010**

10:00-11:30 **Michel Talagrand**

Are many small sets explicitly small?

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11:30-11:50 *Coffee break*

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11:50-12:30 **Stanislaw J. Szarek**

Almost-Euclidean subspaces of  $\ell_1^N$  via tensor products: a low-tech approach to randomness reduction

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12:30-14:30 *Lunch break*

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14:30-15:10 **Holger Rauhut**

Compressive Sensing and Structured Random Matrices

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15:10-15:40 *Coffee break*

---

15:40-16:20 **Charles Dossal**

Support identification of sparse vectors from random noisy measurements

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16:25-17:05 **Franck Barthe**

Convergence of bipartite functionals in many dimensions

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**Thursday, 20.05.2010**

09:45-10:25 **Ronald Devore**

Approximating and Querying Functions in High Dimensions

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10:30-11:10 **Shahar Mendelson**

On weakly bounded empirical processes

---

11:10-11:40 *Coffee break*

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11:40-12:20 **Artem Zvavitch**

The iterations of intersection body operator

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12:25-13:05 **Boris Kashin**

On the uniform approximation of the partial sum of the Dirichlet series by a shorter sum

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13:05-14:30 *Lunch break*

---

14:30-15:10 **Keith Ball**

Noise sensitivity and Gaussian surface area

---

15:10-15:40 *Coffee break*

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15:40-16:20 **Shuheng Zhou**

Thresholded Lasso for High Dimensional Variable Selection and Statistical Estimation

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16:25-17:05 **Krzysztof Oleszkiewicz**

$L^1$ -smoothing for the multi-dimensional Ornstein-Uhlenbeck semigroup

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**Friday, 21.05.2010**

9:45-10:25 **Roman Vershynin**

Estimation of covariance matrices

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10:30-11:10 **Sandrine Péché**

The spectrum of non white sample covariance matrices

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11:10-11:40 *Coffee break*

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11:40-12:20 **Grigoris Paouris**

On the existence of a subgaussian direction on log-concave measures

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12:25-13:05 **Assaf Naor**

Random martingales and localization of maximal inequalities

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## 2 Abstracts

Mon, 05.17.2010., 10:30-11:10

### Vladimir Koltchinskii

Georgia Institute of Technology, Atlanta, USA

#### Sparse Recovery in Linear Spans and Convex Hulls of Infinite Dictionaries

Let  $S$  be an arbitrary measurable space and let  $(X, Y)$  be a random couple in  $S \times \mathbb{R}$ . Denote by  $f^*(x) := \mathbb{E}(Y|X = x)$  the regression function and by  $\Pi$  the distribution of  $X$ . Let  $\mathcal{H}$  be a set of uniformly bounded functions on  $S$  which will be called a dictionary. Suppose that  $\mathcal{H}$  is equipped with a  $\sigma$ -algebra and with a finite measure  $\mu$  and let  $\mathcal{C} \subset L_1(\mu)$  be a convex set of functions on  $\mathcal{H}$ . For  $\lambda \in \mathcal{C}$ ,  $f_\lambda(\cdot) := \int_{\mathcal{H}} h(\cdot)\lambda(h)\mu(dh)$ . Let  $(X_1, Y_1), \dots, (X_n, Y_n)$  be i.i.d. copies of  $(X, Y)$ . We study the following penalized empirical risk minimization problem

$$\hat{\lambda}^\varepsilon := \operatorname{argmin}_{\lambda \in \mathcal{C}} \left[ \frac{1}{n} \sum_{i=1}^n (Y_i - f_\lambda(X_i))^2 + \varepsilon \|\lambda\|_{L_1(\mu)} \right]$$

as well as its versions with somewhat different convex complexity penalties. Our goal is to show (under proper assumptions on the dictionary) that if there exists a “sparse” function  $\lambda \in \mathcal{C}$  (in the sense that it is supported on a relatively “small” subset of the dictionary) such that  $f_\lambda$  provides a “good”  $L_2(\Pi)$ -approximation of the target function  $f^*$ , then, with a high probability, the solution  $\hat{\lambda}^\varepsilon$  is “approximately sparse” and  $f_{\hat{\lambda}^\varepsilon}$  provides a “good” approximation of  $f^*$  (with an error depending on the sparsity of the problem).

This is a joint work with Stas Minsker.

Mon, 05.17.2010., 11:40-12:20

## **Jared Tanner**

University of Edinburgh, Edinburgh, Scotland

### Random matrix theory and stochastic geometry in compressed sensing

Much of the theory developed for compressed sensing is only known to be satisfied for random sensing matrices. We review what is known for Gaussian matrices through the lens of standard eigenvalue random matrix theory as well as stochastic geometry. The currently best known bounds on the restricted isometry constants is presented.

Mon, 05.17.2010., 12:25-13:05

## **Omer Friedland**

Université Pierre et Marie Curie, Paris, France

### Random embedding of $\ell_p^n$ into $\ell_r^N$

For any  $0 < p < 2$  and any integers  $N > n$ , we give an explicit definition of a random operator  $S : \ell_p^n \rightarrow \mathbb{R}^N$  such that for every  $0 < r < p < 2$  with  $r \leq 1$ , the operator  $S_r = S : \ell_p^n \rightarrow \ell_r^N$  satisfies with overwhelming probability that  $\|S_r\| \|(S_r)_{\text{Im} S}^{-1}\| \leq C(p, r)^{n/(N-n)}$ , where  $C(p, r) > 0$  is a real number depending only on  $p$  and  $r$ . One of the main tools that we develop is a new type of multidimensional Esseen inequality for studying small ball probabilities.

Joint work with Olivier Guédon.

Mon, 05.17.2010., 14:30-15:10

## **Philippe Jaming**

Université d'Orléans, Orléans, France

### Some annihilating pairs in harmonic analysis

In this talk, I will survey some results from harmonic analysis concerning the following notion :

*A pair of subsets  $S, \Sigma$  of  $\mathbf{R}$  is annihilating (for the Fourier transform) if the only function with support in  $S$  and spectrum (support of the Fourier transform) in  $\Sigma$  is the zero function.*

In particular, we will show that some results concerning compressed sensing (e.g. the Uniform Uncertainty Principle of Candès-Tao) can be seen as discrete analogues of the quantitative version of the above notion.

Mon, 05.17.2010., 15:40-16:20

## **Francis Bach**

INRIA ENS, Paris, France

### High-Dimensional Non-Linear Variable Selection

We consider the problem of high-dimensional non-linear variable selection for supervised learning. Our approach is based on performing linear selection among exponentially many appropriately defined positive definite kernels that characterize non-linear interactions between the original variables. To select efficiently from these many kernels, we use the natural hierarchical structure of the problem to extend the multiple kernel learning framework to kernels that can be embedded in a directed acyclic graph; we show that it is then possible to perform kernel selection through a graph-adapted sparsity-inducing norm, in polynomial time in the number of selected kernels. Moreover, we study the consistency of variable selection in high-dimensional settings, showing that under certain assumptions, our regularization framework allows a number of irrelevant variables which is exponential in the number of observations.

Mon, 05.17.2010., 16:25-17:05

## **Radosław Adamczak**

University of Warsaw, Warsaw, Poland

### Geometric properties of random matrices with independent log-concave rows/columns

I will give an overview of results obtained in the last two years with O. Guédon, A. Litvak, A. Pajor and N. Tomczak-Jaegermann, concerning random matrices with independent log-concave rows/columns. In particular I will briefly describe:

- estimates of norms of such matrices,
- the solution of the Kannan-Lovasz-Simonovits question on empirical approximation of the covariance matrix of a log-concave measure,
- the restricted isometry property for matrices with independent columns and its relation to neighbourliness properties of polytopes spanned by independent vectors drawn from a convex body,
- estimates for the smallest singular value and the condition number of a square matrix.

Tues, 05.18.2010., 9:45-10:25

## Leonid Pastur

Mathematical Division, Institute for Low Temperatures, Kharkiv,  
Ukraine

### Central Limit Theorem for Linear Eigenvalue Statistics of Random Matrices with Independent Entries

We consider  $n \times n$  real symmetric and hermitian random matrices  $n^{-1/2}W$  with independent (modulo symmetry condition) entries and the (null) sample covariance matrices  $n^{-1}X^*X$  with independent entries of  $m \times n$  matrix  $X$ . Assuming first that the 4th cumulant (excess)  $\kappa_4$  of entries of  $W$  and  $X$  is zero and that their 4th moments satisfy a Lindeberg type condition, we prove that linear statistics of eigenvalues of the above matrices satisfy the Central Limit Theorem (CLT) as  $n \rightarrow \infty$ ,  $m \rightarrow \infty$ ,  $m/n \rightarrow c \in [0, \infty)$  with the same variance as for Gaussian matrices if the test functions of statistics are smooth enough (essentially of the class  $\mathbb{C}^5$ ). This is done by using a simple "interpolation trick" from the known results for the Gaussian matrices and the integration by parts, presented in the form of certain differentiation formulas. Then, by using a more elaborated version of the techniques, we prove the CLT in the case of non-zero excess of entries again for essentially  $\mathbb{C}^5$  test function. Here the variance of statistics contains an additional term proportional to  $\kappa_4$ . The proofs of all limit theorems follow essentially the same scheme.

Tues, 05.18.2010., 10:30-11:10

## Alexandre Tsybakov

CREST and Université Pierre et Marie Curie, Paris, France

### Estimation of High-Dimensional Low Rank Matrices

We consider the model of trace regression where one observes linear combinations of entries of an unknown  $m \times T$ -matrix  $A$  corrupted by noise. We are particularly interested in the high-dimensional setting where the number  $mT$  of unknown entries can be much larger than the sample size  $N$ . Our aim is to estimate the matrix  $A$  under the assumption that it has small rank. In order to shrink towards a low-rank representation, we consider penalized least squares estimators with a Schatten- $p$  quasi-norm penalty,  $p \leq 1$ . We study these estimators under two possible assumptions - a modified version of the restricted isometry condition and a uniform bound on the ratio “empirical norm induced by the sampling operator/Frobenius norm”. The main results are stated as non-asymptotic upper bounds on the prediction risk and on the Schatten- $q$  risk of the estimators for  $q \in [p, 2]$ . As examples, we consider applications to multi-task learning and matrix completion problems.

This is a joint work with Angelika Rohde.

Tues, 05.18.2010., 11:40-12:20

## Rafał Łatała

Warsaw University, Warsaw, Poland

### On 1-symmetric logarithmically concave distributions

Let  $X = (X_1, X_2, \dots, X_n)$  be a random vector with unconditional logarithmically concave distribution. We will present two-sided estimates for moments of linear combinations of  $X_i$ 's and discuss a related open problem of estimating expected value of suprema of such combination.

Tues, 05.18.2010., 11:25-13:05

## **Ionel Popescu**

Georgia Institute of Technology, Atlanta, USA

### Random Matrices and Analyticity of the Planar Limit

The planar limit is a combinatorial generating function which counts the number of planar graphs of arbitrary valency and this is intimately connected to random matrix calculations. On the other hand an analytic treatment of the random matrix reduces the problem to studying the minimizer of the so called logarithmic energy with external field.

We will show an elementary way of dealing with the minimizer of the logarithmic energy with external fields. This is based on manipulations of Chebyshev polynomials and combinatorial identities which gives a nice formula for the minimum of the energy. This indicates why the analyticity claim of the planar limit holds true. Based on this formula, we are able to compute some planar limits in closed form.

This is joint work with Stavros Garoufalidi.

Wed, 05.19.2010., 10:00-11:30

## **Michel Talagrand**

Université Pierre et Marie Curie, Paris, France

Are many small sets explicitly small?

It might well be that some fundamental general properties of sets in large dimensions remain to be discovered. These properties are formulated through a series of ambitious conjectures.

Wed, 05.19.2010., 11:50-12:30

## **Stanislaw J. Szarek**

Université Pierre et Marie Curie, Paris, France

Almost-Euclidean subspaces of  $\ell_1^N$  via tensor products: a low-tech approach to randomness reduction

It has been known since 1970's that the  $N$ -dimensional  $\ell_1$ -space contains nearly Euclidean subspaces whose dimension is  $\Omega(N)$ . However, proofs of existence of such subspaces were probabilistic, hence non-constructive, which made the results not-quite-suitable for subsequently discovered applications to high-dimensional nearest neighbor search, error-correcting codes over the reals, compressive sensing and other computational problems. In this paper we present a "low-tech" scheme which, for any  $a > 0$ , allows to exhibit nearly Euclidean  $\Omega(N)$ -dimensional subspaces of  $\ell_1^N$  while using only  $N^a$  random bits. Our results extend and complement (particularly) recent work by Guruswami-Lee-Wigderson. Characteristic features of our approach include (1) simplicity (we use only tensor products) and (2) yielding arbitrarily small distortions, or "almost Euclidean" subspaces.

Joint work with P. Indyk.

Wed, 05.19.2010., 14:30-15:10

## **Holger Rauhut**

Universität Bonn, Bonn, Germany

### Compressive Sensing and Structured Random Matrices

Compressive sensing is a new paradigm in signal processing and sampling theory that predicts that sparse signals can be recovered from a small number of linear and non-adaptive measurements using convex optimization. Quite remarkably, all good constructions of the so called measurement matrix known so far are based on randomness. While Gaussian random matrices provide optimal recovery guarantees, such unstructured matrices are of limited use in applications. Indeed, structure often allows to have fast matrix vector multiplies. This is crucial in order to deal with large scale problems. The talk discusses models of structured random matrices that are useful in certain applications, and presents corresponding recovery guarantees. An important type of structured random matrix arises in connection with sampling sparse expansions in terms of bounded orthogonal systems (such as the Fourier system). The second type of structured random matrices to be discussed arises from random circulant matrices, that is, from convolution.

Wed, 05.19.2010., 15:40-16:20

## **Charles Dossal**

Université de Bordeaux 1, Bordeaux, France

### Support identification of sparse vectors from random noisy measurements

It is known for a while that  $\ell_1$  minimization may recover most sparse vectors from random and noisy measurements. The classical approach to explain this fact use Restricted Isometry Properties (RIP). RIP approach ensure that all vectors with  $S = \mathcal{O}(\frac{n}{\log(N/n)})$  non zero components can be recovered by  $\ell_1$  minimization. An alternative approach that does not use RIP is proposed. This new approach is developed for gaussian matrices and ensures that the support of a vector  $x$  can be recover perfectly if  $S \leq \frac{n}{2 \ln N}$ . The differences with RIP results are detailed.

Wed, 05.19.2010., 16:25-17:05

## **Franck Barthe**

Université Paul-Sabatier, Toulouse, France

### Convergence of bipartite functionals in many dimensions

We consider combinatorial optimisation problems for two random samples (bipartite matching being the simplest example) and provide a simple framework for proving almost sure convergence.

Joint work with Charles Bordenave.

Thur, 05.20.2010., 09:45-10:25

## **Ronald Devore**

University of South Carolina, Columbia, USA

### Approximating and Querying Functions in High Dimensions

The approximation of functions in high dimensions suffers from the curse of dimensionality. This means smoothness of the function alone is not sufficient to allow accurate recovery. We shall discuss other models for functions based on variable reduction as well as smoothness and give new algorithms for recovery based on these models.

Thur, 05.20.2010., 10:30-11:10

## **Shahar Mendelson**

Technion, Haifa, Israël

### On weakly bounded empirical processes

We will discuss some new results on the behavior of empirical processes indexed by classes of functions that need not be uniformly bounded. Instead, we will assume that the functions in the class have tails with nice decay properties (e.g. subgaussian or subexponential). We will show that the structure of a typical coordinate projection of the class  $F$ , that is, the random set  $\{(f(X_1), \dots, f(X_N)) : f \in F\} \subset \mathbb{R}^N$  has a well defined geometric structure. We will then explain how this structure can be used to control

$$\sup_{f \in F} \left| \frac{1}{N} \sum_{i=1}^N f^2(X_i) - \mathbb{E}f^2 \right|$$

in situations where contraction based methods will only lead to trivial bounds. Finally, if time permits, we will explain how these results can be used to extend some classical results in Asymptotic Geometric Analysis (e.g. low- $M^*$  estimates and estimates on the norms of certain random matrices) from the gaussian ensemble to any isotropic, log-concave ensemble.

Thur, 05.20.2010., 11:40-12:20

## **Artem Zvavitch**

Kent state university, Kent, USA

### The iterations of intersection body operator

The notion of an intersection body of a star body was introduced by E. Lutwak:  $K$  is called the intersection body of  $L$  if the radial function of  $K$  in every direction is equal to the  $(d-1)$ -dimensional volume of the central hyperplane section of  $L$  perpendicular to this direction.

The notion turned out to be quite interesting and useful in Convex Geometry and Geometric tomography. It is easy to see that the intersection body of a ball is again a ball. E. Lutwak asked if there is any other star-shaped body that satisfy this property. We will present a solution to a local version of this problem: if a convex body  $K$  is closed to a unit ball and intersection body of  $K$  is equal to  $K$ , then  $K$  is a unit ball. We will also discuss a harmonic analysis version of this question which studies the Radon transform of powers of a given function.

Joint work with A. Fish, F. Nazarov and D. Ryabogin.

Thur, 05.20.2010., 12:25-13:05

## **Boris Kashin**

Steklov Mathematical Institute, Moscow, Russia

On the uniform approximation of the partial sum of the Dirichlet series by a shorter sum

Thur, 05.20.2010., 14:30-15:10

## **Keith Ball**

University college London, London, Great Britain

Noise sensitivity and Gaussian surface area

I will discuss the problem of noise sensitivity for Boolean functions that has developed over the last half dozen years as well as the Gaussian equivalent: the Gaussian surface area. I will recall some older results of Nazarov and myself and then explain the recent solution by D. Kane of a problem of Klivans, O'Donnell and Servedio.

Thur, 05.20.2010., 15:40-16:20

## **Shuheng Zhou**

ETH, Zürich, Swiss

Thresholded Lasso for High Dimensional Variable Selection and Statistical Estimation

A line of recent work has demonstrated that sparsity is a powerful technique in signal reconstruction and in statistical estimation. Given  $n$  noisy samples with  $p$  dimensions, where  $n \ll p$ , we show that the multi-step thresholding procedure based on the Lasso, we call it the Thresholded Lasso, can accurately estimate a sparse vector  $\beta$  in  $p$ -dimensional space in a linear model. We show that under the Restricted Eigenvalue (RE) condition (Bickel-Ritov-Tsybakov 09), it is possible to achieve the  $L_2$ -loss within a logarithmic factor of the ideal mean square error one would achieve with an oracle while selecting a sufficiently sparse model (hence achieving "sparse oracle inequalities") the oracle would supply perfect information about which coordinates are non-zero and which are above the noise level.

Thur, 05.20.2010., 16:25-17:05

## Krzysztof Oleszkiewicz

Warsaw University, Warsaw, Poland

### $L^1$ -smoothing for the multi-dimensional Ornstein-Uhlenbeck semigroup

Let  $(T_t)_{t \geq 0}$  be the standard Ornstein-Uhlenbeck semigroup on  $\mathbb{R}^d$  and let  $\gamma_d$  be the standard  $\mathcal{H}(0, Id_d)$  Gaussian measure. We prove that for every  $t > 0$  there exists a function  $\psi_{t,d} : [1, \infty) \rightarrow [1, \infty)$  tending to infinity and such that for every Borel function  $f : \mathbb{R}^d \rightarrow [0, \infty)$  with  $\int_{\mathbb{R}^d} f d\gamma_d = 1$  and every  $u > 1$  there is

$$\gamma_d(\{x \in \mathbb{R}^d : (T_t f)(x) > u\}) \leq \frac{1}{u\psi_{t,d}(u)}$$

(note that with the right hand side replaced by  $1/u$  the inequality would be trivial). This result is an  $L^1$  counterpart of the classical hypercontractive estimate  $\|T_t\|_{L^q(\mathbb{R}^d, \gamma_d) \rightarrow L^p(\mathbb{R}^d, \gamma_d)} \leq 1$  for  $p > q > 1$  and  $t \geq \frac{1}{2}(\ln(p-1) - \ln(q-1))$ . This is a partial confirmation of a conjecture stated by M. Talagrand in the discrete cube setting.

My talk will be based on a work in progress, joint with K. Ball, F. Barthe, W. Bednorz and P. Wolff.

Fri, 05.21.2010., 9:45-10:25

## **Roman Vershynin**

University of Michigan, Ann Arbor, USA

### Estimation of covariance matrices

Consider an unknown distribution on  $\mathbb{R}^n$ . What can we learn about this distribution by sampling  $N$  independent points? More specifically, what is the minimal sample size  $N = N(n)$  that captures the covariance structure of the distribution? Such problems are fundamental in multivariate statistics and many applications (wireless communications, genomics, databases, etc.) They translate into basic questions about the spectra of random matrices with independent columns (formed by the sample vectors). We will see some conjectures and outline recent results in this area.

Fri, 05.21.2010., 10:30-11:10

## **Sandrine Péché**

Université de Grenoble, Grenoble, France

### The spectrum of non white sample covariance matrices

We will review some recent results about the eigenvalues and eigenvectors of large sample covariance matrices. In particular, the problem of estimating the true covariance of a sample in high dimensional setting will be considered.

Fri, 05.21.2010., 11:40-12:20

## Grigoris Paouris

Texas A&M University, College Station, USA

On the existence of a subgaussian direction on log-concave measures

We show that if  $\mu$  is a centered log-concave probability measure on  $\mathbb{R}^n$  then, there exists  $\theta \in S^{n-1}$  such that

$$\mu(\{x \in \mathbb{R}^n : |\langle x, \theta \rangle| \geq ct\mathbb{E}|\langle \cdot, \theta \rangle|\}) \leq e^{-\frac{t^2}{\log(t+1)}}$$

for all  $1 \leq t \leq \sqrt{n \log n}$ , where  $c > 0$  is an absolute constant. This slightly improves the previously known estimates.

This is a joint work with A. Giannopoulos and P. Valettas.

## Assaf Naor

Courant Institut-University of New-York, New-York, USA

### Random martingales and localization of maximal inequalities

Let  $(X, d, \mu)$  be a metric measure space. For  $\emptyset \neq R \subseteq (0, \infty)$  consider the Hardy-Littlewood maximal operator

$$M_R f(x) \stackrel{\text{def}}{=} \sup_{r \in R} \frac{1}{\mu(B(x, r))} \int_{B(x, r)} |f| d\mu.$$

We show that if there is an  $n > 1$  such that one has the “microdoubling condition”  $\mu(B(x, (1 + \frac{1}{n})r)) \lesssim \mu(B(x, r))$  for all  $x \in X$  and  $r > 0$ , then the weak  $(1, 1)$  norm of  $M_R$  has the following localization property:

$$\|M_R\|_{L_1(X) \rightarrow L_{1,\infty}(X)} \asymp \sup_{r > 0} \|M_{R \cap [r, nr]}\|_{L_1(X) \rightarrow L_{1,\infty}(X)}.$$

An immediate consequence is that if  $(X, d, \mu)$  is Ahlfors-David  $n$ -regular then the weak  $(1, 1)$  norm of  $M_R$  is  $\lesssim n \log n$ , generalizing a result of Stein and Strömberg. We show that this bound is sharp, by constructing a metric measure space  $(X, d, \mu)$  that is Ahlfors-David  $n$ -regular, for which the weak  $(1, 1)$  norm of  $M_{(0,\infty)}$  is  $\gtrsim n \log n$ . The localization property of  $M_R$  is proved by assigning to each  $f \in L_1(X)$  a distribution over *random* martingales for which the associated (random) Doob maximal inequality controls the weak  $(1, 1)$  inequality for  $M_R$ .

Joint work with Terence Tao.



### 3 Informations

#### 3.1 How can I get to the conference?

The talks will take place on the campus “Cité Descartes” of Université Paris-Est Marne-la-Vallée. The best way to come to the campus is by RER. Take line A of RER (the red one) direction Marne-la-Vallée/Chessy or Torcy. Get off the train at the station Noisy-Champs (the best is to get off from the station at the head of the train when coming from Paris). The Copernic building is at 5 or 10 minutes walk from the train station Noisy-Champs (cf. the maps below). All the talks will take place in the Auditorium inside Copernic Building just 30 meters on your left from the main entrance (ground floor).

#### 3.2 Public Transport in Paris

In Paris, the company of public transport - bus and metro - is the RATP (<http://www.ratp.fr>).

Bus and Metro tickets are the same. You can buy them in any Metro station and in most of the cafés which sell tobacco. In a bus, you can only buy tickets one by one from the bus driver. The prices are as follows:

- 1,40 Euros for one single ticket
- 10,70 Euros for a ”carnet” (which contains 10 tickets)

You can also buy special tickets. Here is a non-exhaustive list:

- for one day, zone 1-2 (i.e. inside Paris): 5,40 Euros
- for one week, from Monday to Sunday, zone 1-2 (i.e. inside Paris) 15,70 Euros
- the cheapest way to assist to the conference (in zone 4) is to buy a “carnet” (10 tickets) for “Noisy-Champs” (from Paris; this is the same ticket for the way to go and the way back).

For people less than 26 years old, special fares exist.

### 3.3 Where can I find some food?

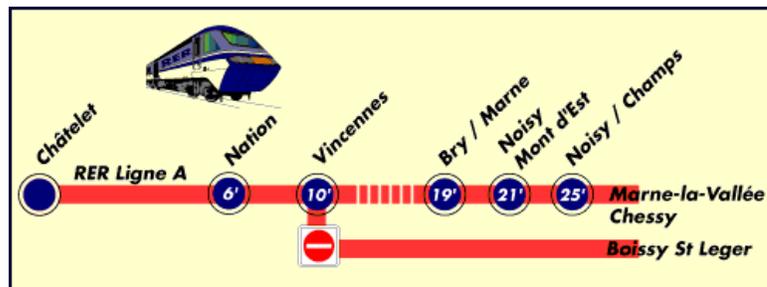
The student restaurant is located on the same level as the auditorum and is accessible to everyone. There is an other restaurant nearby at Ecole Nationale des Ponts et Chaussées that we recommend. It is located in the building facing Copernic building (called “E.N.P.C” on the map below). Restaurants in ENPC is open from 11h30 to 14h.

There will be a buffet on Tuesday organized by the Laboratoire d’Analyse et Mathématiques Appliquées (LAMA).

# MAP: How to get to the conference? (Part I)



Take RER A up to the train station "Noisy-Champs". Direction "Marne-la-Vallée Chessy".



The correct direction is "Marne-la-Vallée Chessy" (or Torcy). Don't take the other direction.

## MAP: How to get to the conference? (Part II)



The conference takes place in the Copernic Building. It is 10 Minutes walk from the train station "Noisy-Champs".

### 3.4 What about internet?

#### **Computers' room:**

The computers' room is located in the Mathematic department of Université Paris-Est Marne-la-Vallée. It is located on the fourth floor of Université Paris-Est Marne-la-Vallée in the Copernic building (same place as the conference). The computer farm is in room 4B110 (without any access code).

#### **Laptops:**

For people having a laptop with **WIFI**, you can connect your computer to the internet via the wireless network of the Mathematic department.

Network: **SEE-PRINTED-BOOKLET**  
Identification: **SEE-PRINTED-BOOKLET**  
password: **SEE-PRINTED-BOOKLET**

## 4 Practical Information

### 4.1 Cafeteria

You will find soft and hot drinks in the cafeteria on the ground floor of Université Paris-Est Marne-la-Vallée. Other cafés can also be found along the way to the RER train station “Noisy-Champs”.

### 4.2 Library

The third floor of the Copernic Building hosts the library of the mathematic department of Université Paris-Est Marne-la-Vallée. The access is by the stairs or by the elevator.

### 4.3 Contact

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<http://perso-math.univ-mlv.fr/users/banach/workshop2010/>

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