Recent Developments for the Singular Values of Skew-Symmetric Gaussian Random Matrices

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\( \mathcal{A} \): The space of \( p \times p \), real, skew-symmetric matrices.

\( A = (a_{ij}) \in \mathcal{A} \): A noncentral Gaussian random matrix with p.d.f.

\[
f(A) = (2\pi)^{-p(p+1)/4} \exp \left[ -\frac{1}{4} \text{tr} \ (A - M)(A - M)' \right],
\]

where \( M = E(A) \).

The singular values of \( A \): \( \sigma_1 > \cdots > \sigma_q > 0 \)
$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$D_\sigma = \begin{cases} \sigma_1 J \oplus \cdots \oplus \sigma_q J, & \text{if } p \text{ is even, } p = 2q \\ \sigma_1 J \oplus \cdots \oplus \sigma_q J \oplus 0, & \text{if } p \text{ is odd, } p = 2q + 1 \end{cases}$

Kuriki (2010) considered the singular value decomposition:

$A = H D_\sigma H'$, where $H \in SO(p)$.

The motivation: Problems in mathematical statistics, and a statistical analysis of a Japanese league’s baseball scores.
Kuriki was led to Harish-Chandra’s integral for $SO(p)$:

$$I_p(\sigma, \nu) = \int_{SO(p)} \exp \left( \frac{1}{2} \text{tr} \ H D_\sigma H' D'_\nu \right) \, dH$$

Note the remarkable connection:

Baseball scores $\longleftrightarrow$ Harish-Chandra’s integral!

My poster will raise open problems concerning the *total positivity* properties of $I_p(\sigma, \nu)$. 