Stationary KPZ Fluctuations For the Stochastic Higher Spin Six Vertex Model

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Random matrices and their applications
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### Stochastic Higher Spin Six Vertex Model [Corwin-Petrov ’15]

**Boltzmann vertex weights**

<table>
<thead>
<tr>
<th></th>
<th>$g$</th>
<th>$g-1$</th>
<th>$g$</th>
<th>$g+1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{L}_{\xi, u}$</td>
<td>$\frac{1-s\xi u q^g}{1-s\xi u}$</td>
<td>$\frac{-s\xi u + s\xi u q^g}{1-s\xi u}$</td>
<td>$\frac{-s\xi u + s^2 q^g}{1-s\xi u}$</td>
<td>$\frac{1-s^2 q^g}{1-s\xi u}$</td>
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</table>

Construct a measure on the set of directed path on $\mathbb{Z}_{\geq 1}^2$

**Map of principal degenerations of the HS6VM**

- HS6VM
- Six Vertex Model
- $q$-TASEP
- Random Polymers
- ASEP
- KPZ

\[ \mathbb{L}_{\xi_x u_t, s_x} \]
In its stationary state the HS6VM can be defined on the full lattice $\mathbb{Z}^2$

Stationary product measure

$$P(m(x, t)) = M \propto \left( \frac{\rho}{s_x \xi_x} \right)^M \frac{(s_x^2, q)_M}{(q, q)_M}$$

An important observable is the stationary height $H$

$$H(x, t) - H(x + \Delta x, t) = \# \text{ of paths in } [x, x + \Delta x] \text{ at time } t,$$

$$H(x, t + \Delta t) - H(x, t) = \# \text{ of paths crossing } x$$

during the time interval $[t, t + \Delta t]$.

Exact formulas for the statistics of $H$

are a consequence of

- Yang Baxter equation

$$L \frac{u_1}{v_2 \sqrt{q}} \frac{1}{\sqrt{q}} * L_{u_1, s} * L_{u_2, s} = L_{u_1, s} * L_{u_2, s} * L \frac{u_1}{v_2 \sqrt{q}} \frac{1}{\sqrt{q}}$$

- Elliptic determinants

$$\frac{\Theta(\xi A/\mathbb{Z}) \prod_{1 \leq i < j \leq n} \Theta(a_i/a_j) \Theta(z_i/z_j)}{\Theta(\xi) \prod_{i,j=1}^n \Theta(a_i/z_j)} = \det_{i,j=1}^n \left( \frac{\Theta(a_i/z_j)}{\Theta(\xi) \Theta(a_i/z_j)} \right)$$
We obtain

\[
\left\langle \frac{1}{(\zeta q^{H(x,t)}, q)_\infty} \right\rangle
\]

\[
= \frac{1}{(q; q)_\infty} \sum_{k \geq 0} \frac{(-1)^k q^{(k)}_2}{(q; q)_k} \det \left( 1 - f \zeta q^{-k}A \right) G(\zeta q^{-k}),
\]

with

\[
f(n) = \frac{1}{1 - q^n / \zeta},
\]

\[
A(n, m) = \frac{1}{(2\pi i)^2} \int_D \frac{dw}{w} \int_C dz \frac{z^m}{w^n} \exp\{xg(z)\} \frac{(q^\frac{x}{w}; q)_\infty}{(q^\frac{x}{z}; q)_\infty} \frac{1}{z-w},
\]

and \( G \) has a more complicated expression.

Our formulas are good for asymptotic analysis!

\[
\frac{H(x, \kappa x) - \eta x}{\gamma x^{1/3}} \xrightarrow{\mathcal{D}} F_{BR}.
\]

Here \( F_{BR} \) is the Baik-Rains distribution [Baik-Rains’00].